

MIND MAP : LEARNING MADE SIMPLE CHAPTER - 5

Continuity and Differentiability

Suppose f is a real function on a subset of the real numbers and let ' c ' be a point in the domain of f . Then f is continuous at c if $\lim_{x \rightarrow c} f(x) = f(c)$
A real function f is said to be continuous if it is continuous at every point in the domain of f . For eg: The function $f(x) = \frac{1}{x}, x \neq 0$ is continuous
Let ' C ' be any non-zero real number, then $\lim_{x \rightarrow c} f(x) \lim_{x \rightarrow c} \frac{1}{x} = \frac{1}{c}$. For $c = 0, f(c) = \frac{1}{c}$ So $\lim_{x \rightarrow c} f(x) = f(c)$ and hence f is continuous at every point in the domain of f .

Suppose f and g are two real functions continuous at a real number c , then, $f+g, f-g, f \cdot g$ and $\frac{f}{g}$ are continuous at $x = c$ ($g(c) \neq 0$).

Suppose f is a real function and c is a point in its domain. The derivative of f at c is $f'(c) = \lim_{h \rightarrow 0} \frac{f(c+h) - f(c)}{h}$
Every differentiable function is continuous, but the converse is not true.

if $f = v \circ u, t = u(x)$ and if both $\frac{dt}{dx}, \frac{dv}{dt}$ exists, then $\frac{df}{dx} = \frac{dv}{dt} \cdot \frac{dt}{dx}$.

Let $y = f(x) = [u(x)]^{v(x)}$
 $\log y = v(x) \log [u(x)]$
 $\frac{1}{y} = v(x) \frac{1}{u(x)} u'(x) + v'(x) \log [u(x)]$
 $\frac{dy}{dx} = y \left[\frac{v(x)}{u(x)} u'(x) + v'(x) \log [u(x)] \right]$
For e.g. : Let $y = a^x$ Then $\log y = x \log a$
 $\frac{1}{y} \cdot \frac{dy}{dx} = \log a$
 $\frac{dy}{dx} = y \log a = a^x \log a$.

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| (i) $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$ | (ii) $\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$ |
| (iii) $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$ | (iv) $\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$ |
| (v) $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{1-x^2}}$ | (vi) $\frac{d}{dx}(\operatorname{cosec}^{-1} x) = -\frac{1}{x\sqrt{1-x^2}}$ |
| (vii) $\frac{d}{dx}(e^x) = e^x$ | (viii) $\frac{d}{dx}(\log x) = \frac{1}{x}$ |

Let $x = f(t), y = g(t)$ be two functions of parameter ' t '.

Then, $\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$ or $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \left(\frac{dx}{dt} \neq 0 \right)$

Thus, $\frac{dy}{dx} = \frac{g'(t)}{f'(t)} \left[\text{provided } f'(t) \neq 0 \right]$

For eg : if $x = a \cos \theta, y = a \sin \theta$ then $\frac{dx}{d\theta} = -a \sin \theta$ and $\frac{dy}{d\theta} = a \cos \theta$, and so $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = -\frac{a \cos \theta}{a \sin \theta} = -\cot \theta$.

Let $y = f(x)$ then $\frac{dy}{dx} = f'(x)$, if $f'(x)$ is differentiable, then $\frac{d}{dx} \left(\frac{dy}{dx} \right) = \frac{d}{dx} f'(x)$ i.e., $\frac{d^2 y}{dx^2} = f''(x)$ is the second order derivative of y w.r.t. x .
For eg : if $y = 3x^2 + 2$, then $y' = 6x$ and $y'' = 6$.

if $f : [a, b] \rightarrow \mathbb{R}$ is continuous on $[a, b]$ and differentiable on (a, b) . Such that $f(a) = f(b)$, then $\exists$ some c in (a, b) s.t. $f'(c) = 0$.

if $f : [a, b] \rightarrow \mathbb{R}$ continuous on $[a, b]$ and differentiable on (a, b) .
Then $\exists$ some c in (a, b) such that $f'(c) = \frac{f(b) - f(a)}{b - a}$
e.g. Let $f(x) = x^2$ defined in the interval $[2, 4]$. Since $f(x) = x^2$ is continuous in $[2, 4]$ and differentiable in $(2, 4)$ as $f'(x) = 2x$ defined in $(2, 4)$. So,
 $f'(c) = \frac{f(b) - f(a)}{b - a} = \frac{16 - 4}{4 - 2} = 6, c \in (2, 4)$.

